

Received 08.12.2017  
Reviewed 11.01.2018  
Accepted 31.01.2018A – study design  
B – data collection  
C – statistical analysis  
D – data interpretation  
E – manuscript preparation  
F – literature search

# Chezy's resistance coefficient in an egg-shaped conduit

Imed LOUKAM<sup>1) BCDE</sup> ✉, Bachir ACHOUR<sup>2) A</sup>, Lakhdar DJEMILI<sup>1) F</sup>

<sup>1)</sup> University of Badji Mokhtar, Faculty of Engineering Science, Department of Hydraulics, BP 12, 23000 Annaba, Algeria;  
e-mail: [i\\_loukam@hotmail.com](mailto:i_loukam@hotmail.com), [l\\_djemili@hotmail.com](mailto:l_djemili@hotmail.com)

<sup>2)</sup> University of Biskra, Research Laboratory in Subterranean and Surface Hydraulics (LARHYSS), Biskra, Algeria;  
e-mail: [bachir.achour@larhyss.net](mailto:bachir.achour@larhyss.net)

**For citation:** Loukam I., Achour B., Djemili L. 2018. Chezy's resistance coefficient in an egg-shaped conduit. Journal of Water and Land Development. No. 37 p. 87–96. DOI: 10.2478/jwld-2018-0028.

## Abstract

When calculating uniform flows in open conduits and channels, Chezy's resistance coefficient is not a problem data and its value is arbitrarily chosen. Such major disadvantage is met in all the geometric profiles of conduits and channels. Knowing the value of this coefficient is essential to both the design of the channel and normal depth calculation. The main objective of our research work is to focus upon the identification of the resistance coefficient relationship. On the basis of the rough model method (RMM) for the calculation of conduits and channels, a general explicit relation of the resistance coefficient in turbulent flow is established with different geometric profiles, particularly the egg-shaped conduit. Chezy's resistance coefficient depends strongly on the filling rate, the discharge, the longitudinal slope, the absolute roughness of the internal walls of the conduit and the kinematic viscosity of the liquid. Moreover, in this work, a simplified method is presented to determine Chezy's resistance coefficient with a limited number of data, namely the discharge, the slope of the conduit, the absolute roughness and the kinematic viscosity. Last but not least, after studying the variation of Chezy's resistance coefficient as a function of the filling rate, an equally explicit expression is given for the easy calculation of this coefficient when its maximum value is reached. Examples of calculation are suggested in order to show how the Chezy's coefficient can be calculated in the egg-shaped conduit.

**Key words:** Chezy's resistance coefficient, egg-shaped conduit, maximum resistance coefficient, rough model method, simplified method, uniform flow

## INTRODUCTION

Determining Chezy's resistance coefficient  $C$  formula since its first appearance in 1775 in its classical form [CARLIER 1972; CHOW 1973; FRENCH 1986] has become the concern of many authors and researchers who have constantly developed appropriate expressions for this parameter characterizing the open channel uniform flow in artificial channels and conduits of sewage or in the river channels and canals of irrigation system [KADBHANE, MANEKAR 2017; REMINI 2016; REMINI *et al.* 2012].

Among the first reflections which have resulted in the expression of the Chezy's coefficient, Prony's suggestion can be highlighted, which suggests the following formula [CARLIER 1972]:

$$\frac{1}{c^2} = \frac{0.000044}{v} + 0.000309 \quad (1)$$

Where:  $V$  = flow velocity.

Tadini simplified Chezy's coefficient by assigning a constant value equal to 50 [CARLIER 1972]. However, BAZIN [1897] proposed an expression for the coefficient  $C$  as a function of the hydraulic radius and

the conduct roughness coefficient whose values are tabulated [CARLIER 1972].

In 1869–1870, GANGUILLET and KUTTER [1869] gave another formula using more parameters such as the hydraulic radius, the roughness coefficient and the longitudinal slope. This formula is expressed through an international unit system where the values of the roughness coefficient are tabulated. The experience has shown that the application of this formula detects a reserve with respect to low slopes (practically less than 0.0001) [CARLIER 1972].

In a simpler form Kutter pointed out another formula very much used in sanitation tunnels [CARLIER 1972] and easier to be applied contrary to that given by Ganguillet. From these expressions one can see that none of them took account of the kinematic viscosity of the flowing liquid.

MANNING [1895] proposed another formula of  $C$  as a function of the hydraulic radius  $R_h$  and the roughness coefficient  $n$ , the latter parameter being the same as that encountered in the Ganguillet–Kutter formula, or:

$$C = \frac{1}{n} R_h^{1/6} \quad (2)$$

Taking into account the Reynolds number  $Re$  defining the flow regime in the conduit, in addition to the absolute roughness  $\varepsilon$  and the hydraulic radius  $R_h$ , Thijssse in 1949 implicitly expressed Chezy's coefficient in the following formula [CARLIER 1972]:

$$C = -18 \log \left[ \frac{\varepsilon}{12R_h} + \frac{C}{3Re} \right] \quad (3)$$

In this relation, the implicit coefficient  $C$  depends upon several hydraulic parameters, namely, the absolute roughness, a practically measurable parameter of the Reynolds number. This latter depends on the kinematic viscosity and obviously on the hydraulic radius.

In 1950, POWELL [1950] expressed the Chezy's coefficient as a way where the hydraulic radius is given in units of feet (Ft) drawing upon the works of KEULEGAN [1938], with an implicit formula resembling that of Thijssse.

The determination of the coefficient  $C$  by Thijssse relation and that of Powell necessitates an iterative calculation method.

SWAMEE and RATHIE [2004] proposed a new general formula (4) for Chezy's coefficient  $C$  which resembles to the Colebrook formula for tapping pipelines with the aim of having an expression that takes into account all the parameters of the flow:

$$C = -2.457 \sqrt{g} \ln \left[ \frac{\varepsilon}{12R_h} + \frac{0.221\nu}{R_h \sqrt{g} R_h} \right] \quad (4)$$

Where:  $\nu$  = the kinematic viscosity;  $g$  = the acceleration due to gravity.

This formula applies to all conduit shapes and in all turbulent flow conditions whether smooth, rough or of transition. However, it may have the disadvantage of being implicit in the case where the linear dimension of the conduit is not given.

Subsequently, several works were carried out in order to determine Chezy's resistance coefficient. We can cite the expressions of STREETER [1936], PERRY *et al.* [1969], MARONE [1970], PYLE, NOVAK [1981], NATOR *et al.* [1996], EAD *et al.* [2000], GIUSTOLISI [2004].

The results of these studies were not really convincing, especially for artificial channels. To that end, in order to enrich the bibliography, the present work aims at formulating an easy-to-use expression for the calculation of Chezy's coefficient  $C$ . Based on the rough model method (RMM) [ACHOUR 2014; 2015a, b, c; ACHOUR, BEDJAOUI 2006; 2012; ACHOUR, SEHTAL 2014] used for calculating conduits and channels, a general relation of resistance coefficient in its explicit form can be established, taking into account the required hydraulic parameters, namely the filling rate, the discharge, the longitudinal slope, the absolute roughness of the internal walls of the conduit and the kinematic viscosity of the liquid. This relation is valid for all states of the turbulent flow in the egg-shaped conduit (Fig. 1).

## GEOMETRICAL CHARACTERISTICS OF EGG-SHAPED CONDUIT

The uniformly flowing new egg-shaped conduit profile (Fig. 1) is often used much like other artificial profiles for the drainage of sewage and storm water collectors of sanitation networks [GÓRSKI *et al.* 2016]. It is geometrically defined by the following elements [ACHOUR 2007]:

- semicircle ( $C_1$ ), of center  $O_1$  and diameter  $D$ ;
- (AB), an arc of the circle ( $C_2$ ) of center  $O_2$  and diameter  $0.25D$ ;
- (AE), an arc of the circle of center  $O'$  and diameter  $8D/3$ ;
- (BF), an arc of the circle of center  $O$  and diameter  $8D/3$ ;
- $G_1G_2 = 0.25D$ ,  $y = 0.034482759D$ ,  $y_m = 1.5D$ .

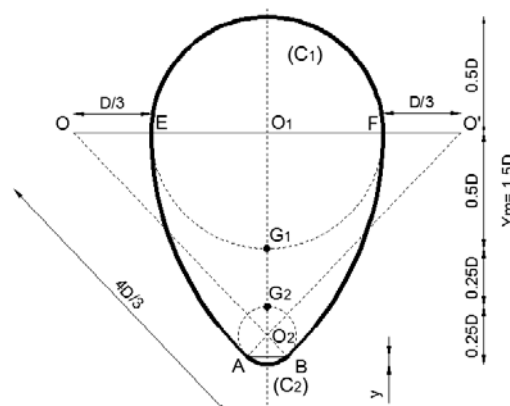


Fig. 1. Egg-shaped conduit profile; source: own elaboration

## HYDRAULIC CHARACTERISTICS

In the conduit (Fig. 1), three cases of studies can be presented according to the position of the normal

depth  $y_n$ . The hydraulic characteristics of the three flow zones, namely: the wetted perimeter  $P$ , the wetted cross-sectional area  $A$  and the hydraulic radius  $R_h$ , are expressed as a function of the filling rate  $\eta = y_n / 1.5D$ .

The flow occupies three distinct spaces defined by the value of  $\eta$ :  $\eta \leq 0.023$ ;  $0.023 \leq \eta \leq 2/3$ ;  $2/3 \leq \eta \leq 1$  [ACHOUR 2007].

**Case 01:**  $\eta \leq 0.023$

$$P = \frac{1}{4} D \cdot \sigma(\eta) \quad \text{[FRENCH 1986]} \quad (5)$$

$$A = \frac{D^2}{64} \sigma(\eta) \varphi(\eta) \quad \text{[FRENCH 1986]} \quad (6)$$

$$R_h = \frac{D}{16} \varphi(\eta) \quad \text{[FRENCH 1986]} \quad (7)$$

Where:

$$\sigma(\eta) \cos^{-1}(1 - 12\eta) \quad (8)$$

$$\varphi(\eta) = 1 - \frac{(1-12\eta)\sqrt{24\eta(1-6\eta)}}{\cos^{-1}(1-12\eta)} \quad (9)$$

**Case 02:**  $0.023 \leq \eta \leq 2/3$

$$P = \rho(\eta)D \quad \text{[FRENCH 1986]} \quad (10)$$

$$A = \frac{8}{9} D^2 \chi(\eta) \quad \text{[FRENCH 1986]} \quad (11)$$

$$R_h = \frac{8}{9} D \frac{\chi(\eta)}{\rho(\eta)} \quad \text{[FRENCH 1986]} \quad (12)$$

Where:

$$\rho(\eta) = \left[ -\frac{8}{3} \sin^{-1} \frac{3}{4} \left( 1 - \frac{3}{2} \eta \right) + 2.350 \right] \quad (13)$$

$$\chi(\eta) = 0.813 - 2 \sin^{-1} \frac{3}{4} \left( 1 - \frac{3}{2} \eta \right) + \frac{3}{2} \left( 1 - \frac{3}{2} \eta \right) \left[ \sqrt{1 - \frac{9}{16} \left( 1 - \frac{3}{2} \eta \right)^2} - \frac{5}{4} \right] \quad (14)$$

**Case 03:**  $2/3 \leq \eta \leq 1$

$$P = 4D \tau(\eta) \omega(\eta) \quad \text{[FRENCH 1986]} \quad (15)$$

$$A = \frac{1}{4} D^2 \tau(\eta) \lambda(\eta) \quad \text{[FRENCH 1986]} \quad (16)$$

$$R_h = \frac{1}{16} \frac{\lambda(\eta)}{\omega(\eta)} D \quad \text{[FRENCH 1986]} \quad (17)$$

Where:

$$\omega(\eta) = \left[ \frac{0.587}{\sin^{-1}(3\eta-2)} + \frac{1}{4} \right] \quad (18)$$

$$\tau(\eta) = \sin^{-1}(3\eta - 2) \quad (19)$$

$$\lambda(\eta) = \left[ 1 + \frac{(3\eta-2)\sqrt{1-(3\eta-2)^2} + 2.890}{\sin^{-1}(3\eta-2)} \right] \quad (20)$$

## GENERAL EXPRESSION OF CHEZY'S RESISTANCE COEFFICIENT

The uniform open channel flow is often governed by the usual formulas such as Chezy's expressing the discharge  $Q$  as suggested below:

$$Q = CA\sqrt{R_h i} \quad (21)$$

The design of the open channel flow draws upon the discharge  $Q$ , the longitudinal slope  $i$ , the filling rate  $\eta$ , the absolute roughness  $\varepsilon$  of the internal wall of the conduit, and the kinematic viscosity  $\nu$  of the liquid. However, the resistance coefficient to flow  $C$  in the formula (21) varies as a function of the filling rate  $\eta$ . This means that this coefficient is not a known data of the problem. It becomes, therefore, the objective of our study.

The relationship of the discharge of ACHOUR and BEDJAOUI [2006] valid in all geometric profiles and established in all turbulent flow regimes (smooth turbulent, transitional and turbulent rough) can show that Chezy's coefficient  $C$  is variable according to all flow parameters.

The discharge  $Q$ , according to ACHOUR and BEDJAOUI [2006], is given by:

$$Q = -4\sqrt{2g} A \sqrt{R_h i} \log \left[ \frac{\varepsilon}{14.8R_h} + \frac{10.04}{\text{Re}} \right] \quad (22)$$

where:  $\varepsilon$  = the absolute roughness characterizing the state of the internal wall of the conduit;  $\text{Re}$  = Reynolds number expressed by the Equation:

$$\text{Re} = 32\sqrt{2} \frac{\sqrt{g i R_h^3}}{\nu} \quad (23)$$

By comparing the two Equations (21) and (22), the Chezy's coefficient  $C$  can be written as:

$$C = -4\sqrt{2g} \log \left[ \frac{\varepsilon}{14.8R_h} + \frac{10.04}{\text{Re}} \right] \quad (24)$$

The Equation (24) reveals that Chezy's resistance coefficient  $C$  depends on the absolute roughness, the hydraulic radius  $R_h$  and the Reynolds number  $\text{Re}$ . This last parameter, according to the Equation (23), is a function of the slope  $i$ , the kinematic viscosity  $\nu$  of the liquid and the hydraulic radius  $R_h$ . The Equations (7), (12) and (17) are taken into account where the hydraulic radius  $R_h$  depends on the filling rate  $\eta$  and the diameter  $D$  of the conduit.

In dimensional terms, the Equation (24) becomes:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2} \log \left[ \frac{\varepsilon}{14.8R_h} + \frac{10.04}{\text{Re}} \right] \quad (25)$$

**Case 01:**  $\eta \leq 0.023$

According to (23) and (7), we have:

$$\text{Re} = \frac{\sqrt{2}}{2} \frac{\sqrt{g i D^3}}{\nu} \varphi(\eta)^{3/2} \quad (26)$$

In the full state of the conduit where  $\eta = 1$ , and from the Equations (23) and (17), we can write:

$$\text{Re}_f = 6.865 \frac{\sqrt{g i D^3}}{\nu} \quad (27)$$

The index  $f$  denotes the full state of the conduit.

So we can also write from Equations (26) and (27):

$$\text{Re} = \text{Re}_f \left( \frac{\varphi(\eta)}{4.551} \right)^{3/2} \quad (28)$$

From the both Equations (7) and (28), the Equation (25) can be rewritten as follows:

$$\frac{c}{\sqrt{g}} = -4\sqrt{2} \log \left[ \frac{\varepsilon/D}{0.925\varphi(\eta)} + \frac{97.463}{\text{Re}_f \varphi(\eta)^{3/2}} \right] \quad (29)$$

**Case 02:**  $0.023 \leq \eta \leq 2/3$

From Equations (23) and (12), we have:

$$\text{Re} = \frac{1024}{27} \frac{\sqrt{g} i D^3}{\nu} \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2} \quad (30)$$

Thus, we can write from both expressions (30) and (27):

$$\text{Re} = 5.525 \text{Re}_f \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2} \quad (31)$$

From (12) and (31), (25) can be rewritten as follows:

$$\frac{c}{\sqrt{g}} = -4\sqrt{2} \log \left[ \frac{\varepsilon/D}{13.16 \frac{\chi(\eta)}{\rho(\eta)}} + \frac{1.817}{\text{Re}_f \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2}} \right] \quad (32)$$

**Case 03:**  $2/3 \leq \eta \leq 1$

According to (23) and (17), we have:

$$\text{Re} = \frac{\sqrt{2}}{2} \frac{\sqrt{g} i D^3}{\nu} \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2} \quad (33)$$

So we can also write from Equations (33) and (27):

$$\text{Re} = \text{Re}_f \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2} \quad (34)$$

From (17) and (34), the Equation (25) can be rewritten as follows:

$$\frac{c}{\sqrt{g}} = -4\sqrt{2} \log \left[ \frac{\varepsilon/D}{0.925 \frac{\lambda(\eta)}{\omega(\eta)}} + \frac{97.463}{\text{Re}_f \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2}} \right] \quad (35)$$

According to Equations (29), (32) and (35), the relative roughness  $\varepsilon/D$ , the filling rate  $\eta$  of the conduit and the Reynolds number corresponding to the full state of the conduit  $\text{Re}_f$  are necessary for Chezy's resistance coefficient  $C$ . When these parameters are known, the Equations (29), (32) and (35) allow the explicit determination of the same coefficient.

### CALCULATION OF CHEZY'S RESISTANCE COEFFICIENT USING THE ROUGH MODEL METHOD (RMM)

The diameter  $D$  can be excluded from the known parameters of the problem in order to compute Chezy's resistance coefficient. These parameters are the discharge, the conduit filling rate, the longitudinal slope  $i$ , the absolute roughness  $\varepsilon$  and the kinematic viscosity  $\nu$  of the flowing liquid. The Equations (29), (32) and (35) will no longer be used for the explicit calculation of Chezy's coefficient  $C$ . So, in this situation, the rough model method (RMM) can be useful to determine this coefficient.

The rough model is particularly characterized by  $\bar{\varepsilon}/\bar{D}_h = 0.037$  [ACHOUR 2007] as the arbitrarily assigned relative roughness value, where  $\bar{D}_h$  is the hydraulic diameter. The chosen relative roughness value

is so large that the prevailed flow regime is fully rough. Thus, the friction factor is  $\bar{f} = 1/16$  according to Colebrook-White relationship for  $\text{Re} = \bar{\text{Re}}$  tending to infinitely large value. For  $\text{Re}$  tending to infinity, Colebrook-White's relationship leads to the Nikuradse formula as follows [ACHOUR 2007]:

$$\bar{f} = \left[ -2 \log \left( \frac{\bar{\varepsilon}/\bar{D}_h}{3.7} \right) \right]^{-2} \quad (36)$$

By inserting the value  $\bar{\varepsilon}/\bar{D}_h = 0.037$ , one can write:

$$\bar{f} = \left[ -2 \log \left( \frac{0.037}{3.7} \right) \right]^{-2} = 4^{-2} = 1/16 \quad (37)$$

In the reference rough model [ACHOUR 2007] the Chezy's coefficient  $\bar{C}$  and the coefficient of friction  $\bar{f} = 1/16$  are linked by the Equation:

$$\bar{C} = \sqrt{8g/\bar{f}} = 8\sqrt{2g} = \text{constant} \quad (38)$$

The rough model is defined by a diameter  $\bar{D}$ , flowing through a discharge liquid  $\bar{Q}$  of kinematic viscosity  $\bar{\nu}$  corresponding to a filling rate  $\bar{\eta}$ , under a longitudinal slope  $\bar{i}$ . For determining Chezy's resistance coefficient characterizing the flow in the conduit, one can assume the following conditions:  $\bar{D} \neq D$ ;  $\bar{Q} = Q$ ;  $\bar{i} = i$ ;  $\bar{\eta} = \eta$ ;  $\bar{\nu} = \nu$ .

**Case 01:**  $\eta \leq 0.023$

From Equations (6) and (7), the Equation (21) becomes

$$Q = \frac{1}{256} \sigma(\eta) \varphi(\eta)^{3/2} \sqrt{C^2 D^5 i} \quad (39)$$

We put:

$$Q^* = \frac{1}{256} \sigma(\eta) \varphi(\eta)^{3/2} \quad (40)$$

Then:

$$Q^* = \frac{Q}{\sqrt{C^2 D^5 i}} \quad (41)$$

According to Equation (41), the relative conductivity of the referential rough model will be:

$$Q^* = \frac{Q}{\sqrt{\bar{C}^2 \bar{D}^5 i}} \quad (42)$$

Or, according to Equation (38), the Equation (42) becomes:

$$\bar{Q}^* = \frac{Q}{\sqrt{128g\bar{D}^5 i}} \quad (43)$$

The Equation (40) is written for referential rough model:

$$\frac{Q}{\sqrt{128g\bar{D}^5 i}} = \frac{1}{256} \sigma(\eta) [\varphi(\eta)]^{3/2} \quad (44)$$

So:

$$\bar{D} = 4 \left[ \sqrt{2} \sigma(\eta) \right]^{-0.4} [\varphi(\eta)]^{-0.6} \left[ \frac{Q}{\sqrt{g} i} \right]^{0.4} \quad (45)$$

The Equation (45) allows the explicit calculation of the diameter  $\bar{D}$  of the referential rough model, knowing that the parameters  $Q$ ,  $i$  and  $\eta$  are known.

The Reynolds number  $\overline{Re}$  characterizing the flow in the referential rough model is, by virtue of Equation (26):

$$\overline{Re} = \frac{\sqrt{2}}{2} \frac{\sqrt{gi\overline{D}^3}}{\nu} [\varphi(\eta)]^{3/2} \quad (46)$$

Or:

$$\overline{Re} = \overline{Re}_f \left(\frac{\varphi(\eta)}{4.551}\right)^{3/2} \quad (47)$$

Where

$$\overline{Re}_f = 6.865 \frac{\sqrt{gi\overline{D}^3}}{\nu} \quad (48)$$

According to the RMM, Chezy's coefficient  $C$  is given as follows [ACHOUR, BEDJAOUI 2006].

$$C = \frac{\overline{c}}{\psi^{5/2}} \quad (49)$$

Where  $\psi$  is a dimensionless parameter determined by the following Equation (ACHOUR, BEDJAOUI [2006; 2012]; ACHOUR, SEHTAL [2014]).

$$\psi = 1.35 \left[ -\log \left( \frac{\varepsilon/\overline{R}_h}{19} + \frac{8.5}{\overline{Re}} \right) \right]^{-\frac{2}{5}} \quad (50)$$

From Equations (7) and (47), the Equation (50) becomes

$$\psi = 1.35 \left[ -\log \left( \frac{\varepsilon/\overline{D}}{1.1875 \varphi(\eta)} + \frac{82.514}{\overline{Re}_f [\varphi(\eta)]^{3/2}} \right) \right]^{-\frac{2}{5}} \quad (51)$$

From (38) and (49):

$$C = \frac{\overline{c}}{\psi^{5/2}} = \frac{8\sqrt{2g}}{\psi^{5/2}} \quad (52)$$

According to the Equation (51), the Equation (52) becomes

$$C = -5.343\sqrt{g} \left[ \log \left( \frac{\varepsilon/\overline{D}}{1.1875 \varphi(\eta)} + \frac{82.514}{\overline{Re}_f [\varphi(\eta)]^{3/2}} \right) \right] \quad (53)$$

The Equation (53) can be written in dimensionless terms as below:

$$\frac{C}{\sqrt{g}} = -5.343 \left[ \log \left( \frac{\varepsilon/\overline{D}}{1.1875 \varphi(\eta)} + \frac{82.514}{\overline{Re}_f [\varphi(\eta)]^{3/2}} \right) \right] \quad (54)$$

**Case 02:**  $0.023 \leq \eta \leq 2/3$

According to the Equations (11) and (12), the Equation (21) becomes

$$Q = \frac{16\sqrt{2}}{27} [\chi(\eta)]^{3/2} [\rho(\eta)]^{-\frac{1}{2}} \sqrt{C^2 D^5 i} \quad (55)$$

We put:

$$Q^* = \frac{16\sqrt{2}}{27} [\chi(\eta)]^{3/2} [\rho(\eta)]^{-\frac{1}{2}} \quad (56)$$

Then:

$$Q^* = \frac{Q}{\sqrt{C^2 D^5 i}} \quad (57)$$

According to Equation (57), the relative conductivity of the referential rough model will be:

$$Q^* = \frac{Q}{\sqrt{C^2 D^5 i}} \quad (58)$$

Or, according to Equation (38), the Equation (58) becomes:

$$\overline{Q}^* = \frac{Q}{\sqrt{128g\overline{D}^5 i}} \quad (59)$$

For referential rough model, the Equation (56) is written:

$$\frac{Q}{\sqrt{128g\overline{D}^5 i}} = \frac{16\sqrt{2}}{27} [\chi(\eta)]^{3/2} [\rho(\eta)]^{-\frac{1}{2}} \quad (60)$$

So:

$$\overline{D} = 0.407 [\rho(\eta)]^{0.2} [\chi(\eta)]^{-0.6} \left[ \frac{Q}{\sqrt{gi}} \right]^{0.4} \quad (61)$$

The Equation (61) allows the explicit calculation of the diameter  $\overline{D}$  of the referential rough model, knowing that the parameters  $Q$ ,  $i$  and  $\eta$  are known. The Reynolds number  $\overline{Re}$  characterizing the flow in the referential rough model is, by virtue of Equation (30):

$$\overline{Re} = \frac{1024}{27} \frac{\sqrt{gi\overline{D}^3}}{\nu} \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2} \quad (62)$$

Let:

$$\overline{Re} = 5.525 \overline{Re}_f \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2} \quad (63)$$

Where:

$$\overline{Re}_f = 6.865 \frac{\sqrt{gi\overline{D}^3}}{\nu}$$

According to Equations (12) and (63), the Equation (50) becomes:

$$\psi = 1.35 \left[ -\log \left( \frac{\varepsilon/\overline{D}}{16.89 \frac{\chi(\eta)}{\rho(\eta)}} + \frac{1.538}{\overline{Re}_f \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2}} \right) \right]^{-\frac{2}{5}} \quad (64)$$

According to (64), the Equation (52) becomes:

$$C = -5.343\sqrt{g} \left[ \log \left( \frac{\varepsilon/\overline{D}}{16.89 \frac{\chi(\eta)}{\rho(\eta)}} + \frac{1.538}{\overline{Re}_f \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2}} \right) \right] \quad (65)$$

Otherwise, in dimensionless terms:

$$\frac{C}{\sqrt{g}} = -5.343 \left[ \log \left( \frac{\varepsilon/\overline{D}}{16.89 \frac{\chi(\eta)}{\rho(\eta)}} + \frac{1.538}{\overline{Re}_f \left[ \frac{\chi(\eta)}{\rho(\eta)} \right]^{3/2}} \right) \right] \quad (66)$$

**Case 03:**  $2/3 \leq \eta \leq 1$

According to the Equations (16) and (17), the Equation (21) becomes:

$$Q = \frac{1}{16} [\tau(\eta)][\omega(\eta)]^{-\frac{1}{2}} [\lambda(\eta)]^{3/2} \sqrt{C^2 D^5 i} \quad (67)$$

We put:

$$Q^* = \frac{1}{16} [\tau(\eta)][\omega(\eta)]^{-\frac{1}{2}} [\lambda(\eta)]^{3/2} \quad (68)$$

Hence:

$$Q^* = \frac{Q}{\sqrt{c^2 D^5 i}} \quad (69)$$

According to Equation (69), the relative conductivity of the referential rough model will be:

$$Q^* = \frac{Q}{\sqrt{c^2 \bar{D}^5 i}} \quad (70)$$

Or, according to Equation (38), the Equation (70) becomes:

$$\bar{Q}^* = \frac{Q}{\sqrt{128g\bar{D}^5 i}} \quad (71)$$

For referential rough model, the Equation (68) is written:

$$\frac{Q}{\sqrt{128g\bar{D}^5 i}} = \frac{1}{16} [\tau(\eta)] [\omega(\eta)]^{-1} [\lambda(\eta)]^{3/2} \quad (72)$$

So:

$$\bar{D} = \left[ \frac{\sqrt{2}}{2} \tau(\eta) \right]^{-0.4} [\omega(\eta)]^{0.2} [\lambda(\eta)]^{-0.6} \left[ \frac{Q}{\sqrt{gi}} \right]^{0.4} \quad (73)$$

The Equation (73) allows the explicit calculation of the diameter  $\bar{D}$  of the referential rough model, if parameters  $Q$ ,  $i$  and  $\eta$  are known. The Reynolds number  $\bar{Re}$  characterizing the flow in the referential rough model is, by virtue of Equation (33):

$$\bar{Re} = \frac{\sqrt{2}}{2} \frac{\sqrt{gi\bar{D}^3}}{v} \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2} \quad (74)$$

Or:

$$\bar{Re} = \bar{Re}_f \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2} \quad (75)$$

$$\text{Where: } \bar{Re}_f = 6.865 \frac{\sqrt{gi\bar{D}^3}}{v}$$

According to Equations (17) and (75), the Equation (50) becomes:

$$\psi = 1.35 \left[ -\log \left( \frac{\varepsilon/\bar{D}}{1.1875 \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]} + \frac{82.514}{\bar{Re}_f \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2}} \right) \right]^{-\frac{2}{5}} \quad (76)$$

According to (76), the Equation (52) becomes:

$$C = -5.343 \sqrt{g} \left[ \log \left( \frac{\varepsilon/\bar{D}}{1.1875 \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]} + \frac{82.514}{\bar{Re}_f \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2}} \right) \right] \quad (77)$$

Otherwise, in dimensionless terms, the Equation (77) can be written as follows:

$$\frac{C}{\sqrt{g}} = -5.343 \left[ \log \left( \frac{\varepsilon/\bar{D}}{1.1875 \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]} + \frac{82.514}{\bar{Re}_f \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2}} \right) \right] \quad (78)$$

### Example 1

For the following data, calculate Chezy's resistance coefficient  $C$  using the rough model method: The discharge  $Q = 0.8 \text{ m}^3 \cdot \text{s}^{-1}$ , the slope  $i = 2 \cdot 10^{-4}$ , the

absolute roughness  $\varepsilon = 10^{-4} \text{ m}$ , the filling rate  $\eta = 0.7$  and the kinematic viscosity  $\nu = 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$ .

**Solution.** For  $\eta = 0.7$ , the calculation will be done by Equations (18), (19), and (20). Thus, with the known parameters  $Q$ ,  $\eta$  and  $i$ , the Equation (73) allows the explicit calculation of the diameter  $\bar{D}$  of the referential rough model.

$\eta = 0.7$ :

$$\omega(\eta) = \left[ \frac{0.587}{\sin^{-1}(3\eta-2)} + \frac{1}{4} \right] = \left[ \frac{0.587}{\sin^{-1}(3 \cdot 0.7 - 2)} + \frac{1}{4} \right] = 6.114$$

$$\tau(\eta) = \sin^{-1}(3\eta - 2) = \sin^{-1}(3 \cdot 0.7 - 2) = 0.1$$

$$\lambda(\eta) = \left[ 1 + \frac{(3\eta-2)\sqrt{1-(3\eta-2)^2+2.890}}{\sin^{-1}(3\eta-2)} \right] = \left[ 1 + \frac{(3 \cdot 0.7 - 2)\sqrt{1-(3 \cdot 0.7 - 2)^2+2.890}}{\sin^{-1}(3 \cdot 0.7 - 2)} \right] = 30.839$$

$\eta = 0.7$ . The Equation (73) gives the diameter  $\bar{D}$ :

$$\begin{aligned} \bar{D} &= \left[ \frac{\sqrt{2}}{2} \tau(\eta) \right]^{-0.4} [\omega(\eta)]^{0.2} [\lambda(\eta)]^{-0.6} \left[ \frac{Q}{\sqrt{gi}} \right]^{0.4} \\ &= \left[ \frac{\sqrt{2}}{2} \cdot 0.1 \right]^{-0.4} 6.114^{0.2} \cdot 30.839^{-0.6} \left[ \frac{0.8}{\sqrt{9.81 \cdot 0.0002}} \right]^{0.4} \\ &= 1.684 \text{ m} \end{aligned}$$

With the Equation (48), we can calculate the Reynolds number  $\bar{Re}_f$  at full state:

$$\begin{aligned} \bar{Re}_f &= 6.865 \frac{\sqrt{gi\bar{D}^3}}{v} = 6.865 \frac{\sqrt{9.81 \cdot 2 \cdot 10^{-4} \cdot 1.684^3}}{10^{-6}} \\ &= 6.65 \cdot 10^5 \end{aligned}$$

The direct calculation of the Chezy's coefficient of resistance to flow  $C$  can be determined without the diameter  $D$  of the tunnel being a data of the problem. This can be done by applying the Equation (77):

$$\begin{aligned} C &= -5.343 \sqrt{g} \left[ \log \left( \frac{\varepsilon/\bar{D}}{1.1875 \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]} + \frac{82.514}{\bar{Re}_f \left[ \frac{\lambda(\eta)}{\omega(\eta)} \right]^{3/2}} \right) \right] \\ &= -5.343 \sqrt{9.81} \left[ \log \left( \frac{10^{-4}/1.684}{1.1875 \frac{30.839}{6.114}} + \frac{82.514}{6.65 \cdot 10^5 \left[ \frac{30.839}{6.114} \right]^{3/2}} \right) \right] \\ &= 78.327 \text{ m}^{0.5} \cdot \text{s}^{-1} \end{aligned}$$

### SIMPLIFIED METHOD

In this part, a simplified method based on the theory of the rough model, is presented to allow the easy determining coefficient  $C$  through a number of limited data with regard to the method already exposed in paragraph 5. For that purpose, the necessary given parameters are only, the discharge, the slope of the conduit, the absolute roughness and the kinematic viscosity of the liquid. So, the calculation of Chezy's coefficient made by this method and that made by the rough model method presented in the paragraph 5, watch, an average relative error lower than 2%. It tried several examples of calculation by making vary the values of the given parameters. If it is assumed

that  $\eta \neq \bar{\eta}$  and when the Equation (73) is applied for rough model, we get:

$$Q^* = \frac{1}{16} [\tau(\bar{\eta})] [\omega(\bar{\eta})]^{-\frac{1}{2}} [\lambda(\bar{\eta})]^{3/2} \quad (79)$$

Where  $Q^*$  is the relative conductivity expressed as follows, according to Equation (71):

$$\bar{Q}^* = \frac{Q}{\sqrt{128 g \bar{D}^5 i}} \quad (80)$$

Let us consider the referential rough model with a diameter  $\bar{D}$  equal to that of the full state of the conduit corresponding to  $\bar{\eta} = 1$ :

At  $\bar{\eta} = 1$ , the Equations (18), (19) and (20) become:  $\omega(\bar{\eta}) = \frac{1.174}{\pi} + \frac{1}{4}$ ,  $\tau(\bar{\eta}) = \frac{\pi}{2}$ ,  $\lambda(\bar{\eta}) = 1 + \frac{5.78}{\pi}$ . Consequently, the Equation (79) leads to  $Q^* = 0.1894\pi$ . For this relative conductivity value, the Equation (79) indicates a second value of the filling rate  $\bar{\eta} \approx 0.8887$  different from  $\bar{\eta} = 1$ .

The hydraulic radius  $\bar{R}_h$  is given according to Equation (17) for the full state of the tunnel:

$$\bar{R}_h = 0.3342\bar{D} \quad (81)$$

For the relative conductivity  $Q^* = 0.1894\pi$ , the diameter  $\bar{D}$  of the full rough model is obtained by the following expression:

$$\bar{D} = (0.1894\pi)^{-0.4} \left( \frac{Q}{\sqrt{128 g i}} \right)^{0.4} \quad (82)$$

The calculation of the Chezy's coefficient  $C$  is made easily by following steps:

- calculate the diameter  $\bar{D}$  corresponding to the full state of the conduit using Equation (82);
- thus, the hydraulic radius  $\bar{R}_h$  is calculated using Equation (81);
- next, the Equation (23) directly calculates the Reynolds number of the rough model;
- therefore, the dimensionless correction factor  $\psi$  is explicitly determined using Equation (50);
- finally, the Chezy's coefficient  $C$  is easily obtained by the Equation (52).

### Example 2

According to the data of example 1 calculate Chezy's coefficient  $C$  using the simplified method:  $Q = 0.8 \text{ m}^3 \cdot \text{s}^{-1}$ ,  $i = 2 \cdot 10^{-4}$ ,  $\varepsilon = 10^{-4} \text{ m}$ ,  $\nu = 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$ .

### Solution

1. By applying the Equation (82), the diameter  $\bar{D}$  of the full rough model is:

$$\begin{aligned} \bar{D} &= (0.1894\pi)^{-0.4} \left( \frac{Q}{\sqrt{128 g i}} \right)^{0.4} = \\ &= (0.1894\pi)^{-0.4} \left( \frac{0.8}{\sqrt{128 \cdot 9.81 \cdot 2 \cdot 10^{-4}}} \right)^{0.4} = 1.484 \text{ m} \end{aligned}$$

2. The hydraulic radius  $\bar{R}_h$  is calculated using the Equation (81).

$$\bar{R}_h = 0.3342\bar{D} = 0.3342 \cdot 1.484 = 0.496 \text{ m}$$

3. The Reynolds number  $\bar{Re}$  of the rough model is calculated from Equation (23):

$$\begin{aligned} \bar{Re} &= 32\sqrt{2} \frac{\sqrt{g i \bar{R}_h^3}}{\nu} = 32\sqrt{2} \frac{\sqrt{9.81 \cdot 2 \cdot 10^{-4} \cdot 0.496^3}}{10^{-6}} = \\ &= 7 \cdot 10^5 \end{aligned}$$

4. Using Equation (50), the calculation of the non-dimensional correction factor is as follows:

$$\begin{aligned} \psi &= 1.35 \left[ -\log \left( \frac{\varepsilon/\bar{R}_h}{19} + \frac{8.5}{\bar{Re}} \right) \right]^{-\frac{2}{5}} \\ \psi &= 1.35 \left[ -\log \left( \frac{10^{-4}/0.496}{19} + \frac{8.5}{7 \cdot 10^5} \right) \right]^{-\frac{2}{5}} = 0.730 \end{aligned}$$

5. The Chezy's coefficient  $C$  is easily computed by Equation (52):

$$C = \frac{8\sqrt{2g}}{\psi^{5/2}} = \frac{8\sqrt{2 \cdot 9.81}}{(0.730)^{5/2}} = 77.83 \text{ m}^{0.5} \cdot \text{s}^{-1}$$

The value of the Chezy's coefficient calculated by the simplified method ( $C_{\text{simplified method}} = 77.83$ ) is less than that calculated in Example 1 by the rough model method ( $C_{\text{MMR}} = 78.327$ ). The relative error rate between these both values is approximately 0.6%.

## MAXIMUM OF CHEZY'S RESISTANCE COEFFICIENT

The Equations (29), (32) and (35) express Chezy's coefficient  $C$  as a function of the dimensionless variables: the filling rate  $\eta$ , the Reynolds number corresponding to the full state of the conduit  $Re_f$  and the relative roughness  $\varepsilon/D$ . According to these relationships, the study of the variation of Chezy's coefficient as a function of the filling rate requires the drawing of the indicative curves for different values of relative roughness and several values of the Reynolds number. Two figures have been made, exposing this variation for both states of turbulent flow in the conduit, one for the smooth state  $\varepsilon/D$  and the second for the rough state  $\varepsilon/D = 0.05$ .

In Figure 2 we can notice curves indicating the variation of  $C/\sqrt{g}$  on the abscissa as a function of  $\eta$  on the ordinate, for fixed Reynolds number values ( $10^4$ ,  $10^5$ ,  $10^6$  and  $10^7$ ). In these curves,  $C/\sqrt{g}$  undergoes an increase along with the filling rate  $\eta$  of the conduit. In a first step, this increase is remarkably rapid whose filling rate varies at the interval  $0 < \eta < 0.3$ . In a second step, where  $\eta > 0.3$ , the increment becomes very slow until the  $C/\sqrt{g}$  takes a maximum value for the same filling rate  $\eta$  in all curves equal to 0.858. Then, due to the decrease in  $C/\sqrt{g}$ , a change in the direction of the curves is shown with the always increasing filling rate up to the full state of the conduit where  $\eta = 1$ . The distinctive feature in Figure 2 is that the curves merge beyond the  $10^5$  value of the Reynolds number, which explains the reason why the vari-

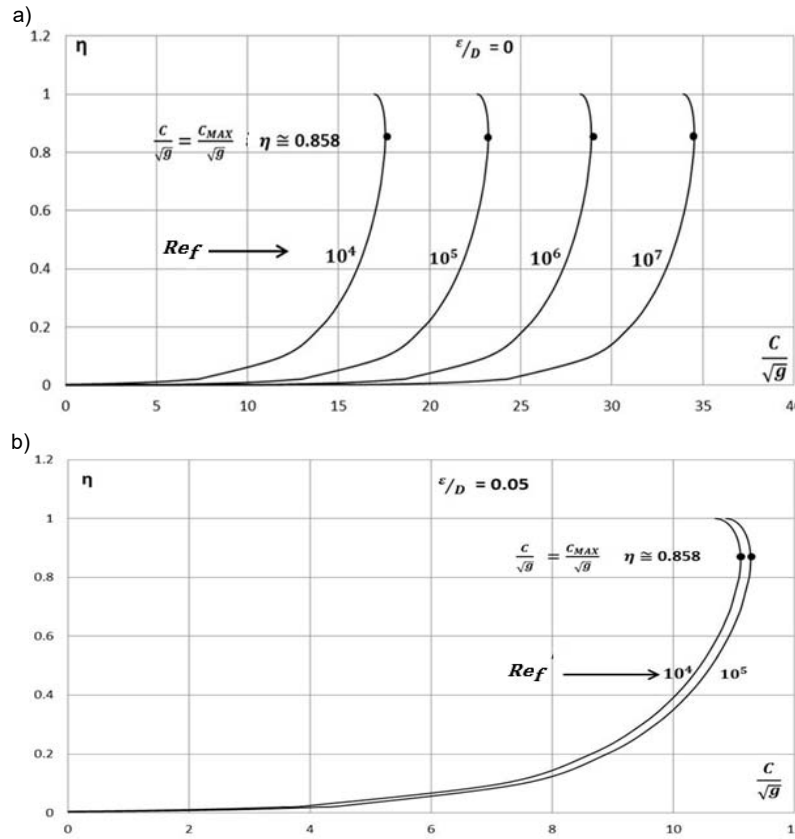


Fig. 2. Variation of  $C/\sqrt{g}$  as a function of the filling rate  $\eta$  according to the equations (29), (32), (35) for fixed values of the relative roughness: a)  $\varepsilon/D = 0$ , b)  $\varepsilon/D = 0.05$  and the Reynolds number  $Re_f$ ; source: own elaboration

ation of  $C/\sqrt{g}$  depends only upon the filling rate  $\eta$  in rough turbulent regime.

For the egg-shaped conduit, Chezy's resistance coefficient reaches a maximum in both figures at the same value of the filling rate  $\eta \cong 0.858$ . This amounts to saying that it does not depend upon the state of the conduit inner wall (rough or smooth), or the value of the Reynolds number.

For this purpose,  $\eta \cong 0.858$

$$\frac{C}{\sqrt{g}} = \frac{C_{max}}{\sqrt{g}}$$

and:

$$\omega(0.858) = \left[ \frac{0.587}{\sin^{-1}(3 \cdot 0.858 - 2)} + \frac{1}{4} \right] = 1.2101$$

$$\lambda(0.858) = \left[ 1 + \frac{(3 \cdot 0.858 - 2)\sqrt{1 - (3 \cdot 0.858 - 2)^2} + 2.890}{\sin^{-1}(3 \cdot 0.858 - 2)} \right] = 6.4958$$

Both values of  $\omega(\eta)$  and  $\lambda(\eta)$  lead to the Equation (35) in the following way:

$$\frac{C_{max}}{\sqrt{g}} = -4\sqrt{2} \log \left[ \frac{\varepsilon/D}{4.965} + \frac{7.8365}{Re_f} \right] \quad (83)$$

or:

$$C_{max} = -4\sqrt{2g} \log \left[ \frac{\varepsilon/D}{4.965} + \frac{7.8365}{Re_f} \right] \quad (84)$$

The latter allows the determining of the maximum of Chezy's resistance coefficient, provided that the two parameters of the relative roughness  $\varepsilon/D$  and the Reynolds number  $Re_f$  are known.

In a contrary case, where the diameter  $D$  of the conduit is not a problem data, the maximum of Chezy's resistance coefficient  $C_{max}$  can be determined by the Equation (78), assigning to the filling rate  $\eta$  the value 0.858.

where:  $\omega(\eta) = 1.2101$ ,  $\lambda(\eta) = 6.4958$ .

Hence, the Equation (78) becomes,

$$\frac{C_{max}}{\sqrt{g}} = -5.343 \log \left( \frac{\varepsilon/D}{6.374} + \frac{6.6345}{Re_f} \right) \quad (85)$$

Let:

$$C_{max} = -5.343\sqrt{g} \log \left( \frac{\varepsilon/D}{6.374} + \frac{6.6345}{Re_f} \right) \quad (86)$$

The above expression permits the maximum of Chezy's resistance without taking into consideration the diameter  $D$  of the conduit.

An example of application is suggested to illustrate the last case treated, describing the steps in detail for the calculation of this coefficient.

### Example 3

For the following data, calculate the maximum value of Chezy's resistance coefficient  $C$ :  $Q = 0.75 \text{ m}^3 \cdot \text{s}^{-1}$

$$i = 2 \cdot 10^{-4}, \quad \varepsilon = 10^{-4} \text{ m}, \quad \eta = 0.6, \quad \nu = 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$$

**Solution.** The calculation for  $\eta = 0.6$ , will be done in equations to the 2<sup>nd</sup> case of flow. Thus, with the known parameters  $Q$ ,  $\eta$  and  $i$ , the Equation (61) allows the explicit calculation of referential rough model diameter  $\bar{D}$ .



For  $\eta = 0.6$ :

$$\rho(0.6) = \left[ -\frac{8}{3} \sin^{-1} \frac{3}{4} \left( 1 - \frac{3}{2} 0.6 \right) + 2.350 \right] = 2.1498$$

$$\begin{aligned} \chi(\eta) &= 0.813 - 2 \sin^{-1} \frac{3}{4} \left( 1 - \frac{3}{2} 0.6 \right) + \\ & - \frac{3}{2} \left( 1 - \frac{3}{2} 0.6 \right) \left[ \sqrt{1 - \frac{9}{16} \left( 1 - \frac{3}{2} 0.6 \right)^2} - \frac{5}{4} \right] \\ &= 0.7008 \end{aligned}$$

The diameter  $\bar{D}$  is given with the Equation (61):

$$\begin{aligned} \bar{D} &= 0.407 [\rho(\eta)]^{0.2} [\chi(\eta)]^{-0.6} \left[ \frac{Q}{\sqrt{g i}} \right]^{0.4} = \\ &= 0.407 \cdot 2.1498^{0.2} \cdot 0.7008^{-0.6} \left( \frac{0.75}{\sqrt{9.81 \cdot 0.0002}} \right)^{0.4} = \\ &= 1.821 \text{ m} \end{aligned}$$

With Equation (48), we can calculate the full state Reynolds number  $\overline{Re}_f$ :

$$\begin{aligned} \overline{Re}_f &= 6.865 \frac{\sqrt{g i \bar{D}^3}}{\nu} = 6.865 \frac{\sqrt{9.81 \cdot 2 \cdot 10^{-4} \cdot 1.821^3}}{10^{-6}} = \\ &= 7.47 \cdot 10^5 \end{aligned}$$

The direct calculation of the maximum of Chezy's resistance coefficient to the flow  $C_{\max}$  is determined without the diameter  $D$  of the conduit being a problem data with the Equation (86):

$$\begin{aligned} C_{\max} &= -5.343 \sqrt{g} \log \left( \frac{\varepsilon/\bar{D}}{6.374} + \frac{6.6345}{\overline{Re}_f} \right) = \\ &= -5.343 \sqrt{9.81} \log \left( \frac{0.0001/1.821}{6.374} + \frac{6.6345}{7.47 \cdot 10^5} \right) = \\ &= 79.608 \text{ m}^{0.5} \cdot \text{s}^{-1} \end{aligned}$$

## CONCLUSIONS

In this study, Chezy's resistance coefficient in the egg-shaped conduit was expressed by several formulas. Taking into account the shape of the studied conduit, determining its geometrical characteristics reveal three cases to be treated, depending on the position of the normal depth of the flowing liquid. General expressions (29), (32) and (35) are obtained from the explicit determination of Chezy's resistance coefficient  $C$  depending on the relative roughness  $\varepsilon/D$ , the filling rate  $\eta$  and the Reynolds number at the full state of the conduit  $\overline{Re}_f$ . However, in the case where the diameter of the conduit is not a problem data, the Equations (53), (65) and (77) are obtained by using the rough model method (RMM) to directly calculate Chezy's resistance coefficient  $C$  as a function of the parameters,  $\varepsilon$ ,  $\bar{D}$ ,  $\eta$  and  $\overline{Re}_f$ .

A simplified method is proposed for the explicit calculation of Chezy's coefficient  $C$ . It is based on (RMM) and has the advantage of being able to use a limited number of data, namely, the discharge  $Q$ , the slope  $i$  of the conduit, the absolute roughness  $\varepsilon$  and the kinematic viscosity  $\nu$ . Finally, a study of Chezy's coefficient variation as a function of the conduit filling rate is carried out by establishing the required curves, assigning fixed values to the relative rough-

ness as well as the Reynolds number in the full state of the conduit. The curves show that Chezy's resistance coefficient reaches a maximum at the filling rate  $\eta \cong 0.858$ . Hence, it is possible to get expressions (84) and (86) for determining the maximum of Chezy's resistance coefficient  $C_{\max}$ , in the conduit diameter, whether it is problem given or not.

The expressions governing the rough model method are derived from the universal relations of Darcy–Weisbach, Colebrook–White and Reynolds, whose validity has been demonstrated in the past. Through these relations, the expression (22) has been established. It gives the exact value of the discharge and is valid throughout the turbulent flow and for all known geometric profiles. The Equation (24), expressing the Chezy's coefficient  $C$ , is derived from the Equation (22). It is therefore valid throughout the turbulent flow and is applicable to all geometric profiles.

The rough model method also relies on the correction coefficient of the linear dimensions  $\psi$ , expressed by the Equation (50). This expression is an excellent approximated relation which gives a maximum error of only 0.4% for a Reynolds number  $Re \geq 2300$ .

## REFERENCES

- ACHOUR B. 2007. Calcul des conduites et canaux par la MMR (Conduites et canaux en charge) [Calculation of pipes and channels by the RMM (Conduits and channels in charge)]. T. 1. Biskra. Larhys Edition Capitale. ISBN 978-9961-9701-0-2 pp. 610.
- ACHOUR B. 2014. Computation of normal depth in parabolic cross sections using the rough model method. Open Civil Engineering Journal. Vol. 8 p. 213–218. DOI 10.2174/1874149501408010213.
- ACHOUR B. 2015a. Analytical solution for normal depth problem in a vertical U-shaped open channel using the rough model method. Journal of Scientific Research and Report. Vol. 6(6) p. 468–475. DOI 10.9734/JSRR/2015/16682.
- ACHOUR B. 2015b. Chezy's resistance coefficient in a circular conduit. Open Civil Engineering Journal. Vol. 9 p. 187–195. DOI 10.2174/1874149501509010187.
- ACHOUR B. 2015c. Chezy's resistance coefficient in a rectangular channel. Journal of Scientific Research and Report. Vol. 7(5) p. 338–347. DOI 10.9734/JSRR/2015/18385.
- ACHOUR B., BEDJAOUI A. 2006. Discussion. Exact solutions for normal depth problem. Journal of Hydraulic Research. Vol. 44(5) p. 715–717. DOI 10.1080/00221686.2006.9521721.
- ACHOUR B., BEDJAOUI A. 2012. Turbulent pipe-flow Computation using the rough model method (RMM). Journal of Civil Engineering and Science. Vol. 1(1) p. 36–41.
- ACHOUR B., SEHTAL S. 2014. The Rough Model Method (RMM). Application to the computation of normal depth in circular conduit. Open Civil Engineering Journal. Vol. 8 p. 57–63. DOI 10.2174/1874149501408010057.
- BAZIN H. 1897. Etude d'une nouvelle formule pour calculer le débit des canaux découverts [Study of a new formula to calculate the discharge of the discovered channels]. Mémoire no. 41. Annales des ponts et chaussées. No. 14(7) p. 20–70.

- CARLIER M. 1972. *Hydraulique générale et appliquée* [General and applied hydraulics]. Paris. Eyrolles pp. 565.
- CHOW V.T. 1973. *Open-channel hydraulics*. New York. McGraw Hill. ISBN 007085906X pp. 680.
- EAD S.A., RAJARATNAM N., KATOPODIS C., ADE F. 2000. Turbulent open-channel flow in circular corrugated culverts. *Journal of Hydraulic Engineering*. Vol. 126(10) p. 750–757. DOI 10.1061/(ASCE)0733-9429(2000)126:10(750).
- FRENCH R.H. 1986. *Open-channel hydraulics*. Civil Engineering Series. New York. McGraw Hill. ISBN 007063424 pp. 705.
- GANGUILLET E., KUTTER W.R. 1869. An investigation to establish a new general formula for uniform flow of water in canals and rivers. *Zeitschrift des österreichischen Ingenieur- und Architekten-Vereines*. H. 21(1) p. 6–25. H. 21(2–3) p. 46–59.
- GIUSTOLISI O. 2004. Using genetic programming to determine Chezy resistance coefficient in corrugated channels. *Journal of Hydroinformatics*. Vol. 6(3) p. 157–173.
- GÓRSKI J., SZELĄG B., BĄK Ł. 2016. Zastosowanie programu SWMM do oceny funkcjonowania oczyszczalni wód deszczowych [The use of the SWMM program to assess the operation of rainwater treatment plants]. *Woda-Środowisko-Obszary Wiejskie*. T. 16. Z. 2 (54) p. 17–35.
- KADBHANE S.J., MANEKAR V.L. 2017. An experimental study on the grape orchard: Effects comparison of two irrigation systems. *Journal of Water and Land Development*. No. 32 p. 41–51. DOI 10.1515/jwld-2017-0005.
- KEULEGAN G.H. 1938. Laws of turbulent flow in open channels. Research paper RP 1151. *Journal of Research, of the National Bureau of Standards*. Vol. 21 p. 707–741.
- MANNING R. 1895. On the flow of water in open channels and pipes. *Transactions of the Institution of Civil Engineers of Ireland*. Vol. 20 p.161–209.
- MARONE V. 1970. Le resistenze al movimento uniforme in unalveo chiuso o aperto di sezione rettangolare e scabrezza definita [Resistances to uniform movement in a closed or open section of rectangular cross-section and defined roughness]. *L'Energia Elettrica*. No. 1 p. 1–20.
- NAOT D., NEZU I., NAKAGAWA H. 1996. Hydrodynamic behavior of partly vegetated open channels. *Journal of Hydraulic Engineering*. Vol. 122(11) p. 625–633. DOI 10.1061/(ASCE)0733-9429(1996)122:11(625).
- PERRY A.E., SCHOFIELD W.H., JOUBERT P.N. 1969. Rough wall turbulent boundary layers. *Journal of Fluid Mechanics*. Vol. 37(2) p. 383–413. DOI 10.1017/S0022112069000619.
- POWELL R.W. 1950. Resistance to flow in rough channels. *Transactions American Geophysical Union*. No. 31(4) p. 575–582. DOI 10.1029/TR031i004p00575/full.
- PYLE R., NOVAK P. 1981. Coefficient of friction in conduits with large roughness. *Journal of Hydraulic Research*. Vol. 19(2) p.119–140.
- REMINI B. 2016. The role of the gallery in the functioning of the foggara. *Journal of Water and Land Development*. No. 29 p. 49–57. DOI 10.1515/jwld-2016-0011.
- REMINI B., ACHOUR B., OULED B.C., BABA A.D. 2012. The Mzab foggara: an original technique for collecting the water rising. *Journal of Water and Land Development*. No. 16 p. 49–53.
- STREETER V.L. 1936. Frictional resistance in artificially roughened pipes. *Transactions of the American Society of Civil Engineers*. No. 101 p. 681–704.
- SWAMEE P.K., RATHIE P.N. 2004. Exact solutions for normal depth problem. *Journal of Hydraulic Research*. Vol. 42(5) p. 543–550.

## Imed LOUKAM, Bachir ACHOUR, Lakhdar DJEMILI

### Współczynnik oporu Chezy'ego w rurach o kształcie jajowatym

#### STRESZCZENIE

Kiedy oblicza się jednorodny przepływ w otwartych rowach i kanałach, współczynnik oporu Chezy'ego nie stanowi problemu, a jego wartość dobiera się arbitralnie. Tę niedogodność spotyka się w przypadku wszystkich profili geometrycznych rur i kanałów. Znajomość współczynnika jest istotna zarówno podczas projektowania kanału, jak i obliczania głębokości. Główny cel pracy skupia się na identyfikacji zależności między współczynnikiem oporu a kształtem rur. Wykorzystując metodę rough model (RMM) ustalono ogólną zależność między współczynnikiem oporu w warunkach turbulencyjnego przepływu a różnymi profilami geometrycznymi, w szczególności jajowatym przekrojem rur. Współczynnik oporu Chezy'ego silnie zależy od tempa napełniania, odpływu, nachylenia wzdłużnego, bezwzględnej szorstkości ścian i lepkości kinematycznej płynu. Ponadto przedstawiono w pracy uproszczoną metodę obliczania współczynnika oporu w warunkach ograniczonej liczby danych, na przykład odpływu, nachylenia rury, bezwzględnej szorstkości czy lepkości kinematycznej. Na końcu, po zbadaniu zmienności współczynnika oporu Chezy'ego w funkcji tempa napełniania, podano wyrażenie służące do łatwego obliczenia tego współczynnika, kiedy osiąga on maksymalną wartość. Sugeruje się przykłady obliczeń, aby pokazać, jak można obliczyć współczynnik Chezy'ego w przekrojach jajowatych.

**Słowa kluczowe:** *maksymalny współczynnik oporu, metoda rough model, metoda uproszczona, przepływ jednorodny, rura o jajowatym przekroju, współczynnik oporu Chezy'ego*