

The calibration of sharp-crested weirs with a horizontal edge used for measuring flows in partially full pipes

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Abstract: This paper presents the results of a laboratory study on the discharge capacity of sharp-crested weirs fitted with a horizontal edge in pipes during open-channel flow conditions and clean water used to measure the outflow. Such sharp-crested weirs are mounted in pipes and are used to control the inflow to separators. The stream profile does not correspond to the profile given by Bazin for sharp crested weirs in channels. A desired location of the water level measurement point for flow rate calculations was provided. Discharge curves were identified for three sharp-crested weirs of 0.0465, 0.0634 and 0.0771 m in height, installed in the pipe of 0.1534 m in diameter and inclinations of 0.5 and 1.0%. The discharge curves for weir flow with free nappe does not show a significant effect of the pipe slope on the weir discharge capacity. The non-dimensional formulas for the discharge capacity of the sharp-crested weir were found as general polynomial regressions. The results indicate that the calibrated sharp-crested weir with a horizontal edge placed in a pipe can be used to control the flow. Due to the scale effect, relationships obtained from the calibration cannot be generalised to other pipe diameters and weirs heights than those analysed.

Keywords: calibration of sharp-crested weirs, discharge characteristics of sharp-crested weirs in pipe; flows in partially full pipes; sharp-crested weirs in pipe

INTRODUCTION

The calibration of flow measuring structures still remains a problem in the design and operation of environmental engineering devices. The use of models in laboratory calibration is relatively easy but burdened with errors due to the omission of variables applicable to the prototype, scale effects, and the extrapolation of results (Vatankhah and Khalili, 2017; Kubrak and Kubrak, 2022). Sharp-crested weirs are usually used for controlling water levels in channels or for accurate measurement of the flow (Bos, 1976). Using sharp-crested weirs for discharge measurements in pipes with free surface flow becomes more and more common in environmental engineering practice, but still it is not well recognised how to apply sharp-crested weirs in outflows with small pipe diameters. Discharge measurement in partially full pipes is a significant practical problem. Oliveto and Hager (1997) used

“Sector Venturi” mounted temporarily in the pipe for discharge measurement in sewers or other free surface pipe flows (Fig. 1). These devices were adapted to highly polluted sewage and pipe diameters larger than 200 mm, when scale effects were negligible. Bettez, Townsend and Comeau (2001) used compound sharp-crested weir for discharge measurements in sewer for pipe diameter of 286 mm. One solution to the problem can be weirs with circular edge described by Addison (1941). Circular weirs are half-circle weirs inserted into conduits and pipes. Circular weirs are independent of the size of the conduit/pipe and typically have minimum requirements in terms of installation (accommodation). V-notch weirs measure small discharge accurately, as small changes in discharge result in large changes in depth (Erickson, Weiss and Gulliver, 2013). Therefore, measurement uncertainty associated with the depth measurement has less effect on the estimated discharge than the other weirs. V-notch

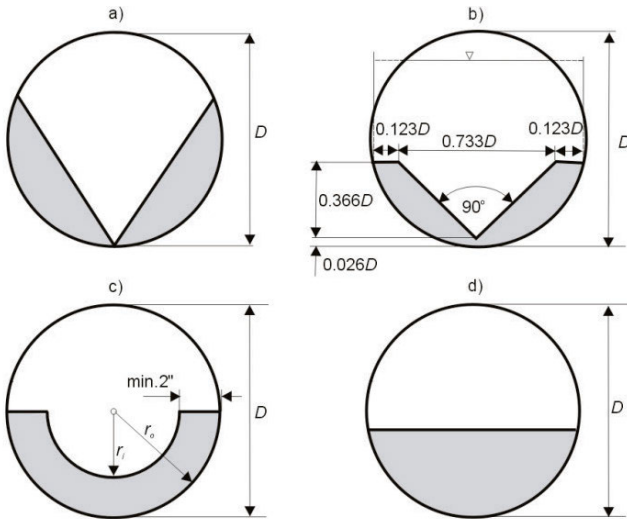


Fig. 1. Sharp-crested weirs used for flow controlling in pipe: a) Sector Venturi (Oliveto and Hager, 1997), b) V-notch weirs (Bettez, Townsend and Comeau, 2001), c) circular weirs (Addison, 1941), d) weirs with horizontal edge; source: own elaboration

weirs have limitations, as large discharge requires more depth compared to other weirs and flumes for the same discharge. Rectangular weirs require less depth for the same discharge than all the other measurement devices shown in Figure 1.

Rectangular weirs do not measure small discharge accurately, as small changes in depth result in major changes at the discharge (Boss, 1976; Erickson, Weiss and Gulliver, 2013).

However, this is not an issue when using horizontal weirs with a sharp crest in separators of petroleum derivative and suspended solids from rainwater. Sharp-crested weirs are mounted in pipes and are used to control the inflow to separators (Fig. 2).

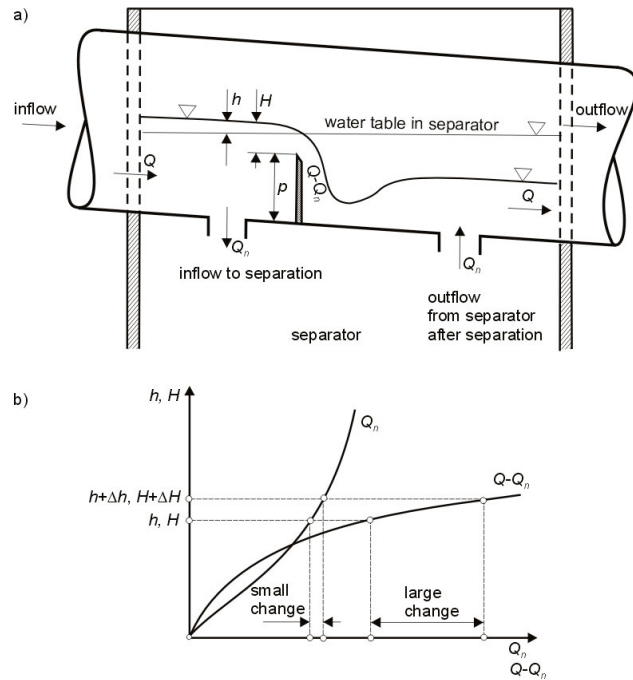


Fig. 2. A control of water level in separators for suspended solids and petroleum derivative using the submerged orifice and sharp-crested weir: a) separator bypass scheme, b) stage-discharge relationships for a sharp-crested weir in a pipe and submerged orifice in a pipe invert; source: own elaboration

The separator is designed for a nominal flow Q_n , which ensures separation of petroleum derivative and suspended solids (Kotowski, 2011). To ensure effective separation with inflows greater than the nominal value ($Q > Q_n$), it must be equipped with a device that directs the inflow difference $Q - Q_n$ out of the separator. For this purpose, the so-called hydraulic bypass is used. As a result, not the entire inflow Q of rainwater is treated, but only its portion which does not exceed the nominal flow $Q \leq Q_n$. This condition can be met by connecting the separator's supply through the submerged orifice and the sharp-crested weir installed in the conduit. It discharges flows greater than the nominal value beyond the separator (Fig. 2). This is possible as a significant increase in the depth of the conduit, resulting from the larger inflow to the separator, causes only a slight increase in the inflow to the separator through the submerged orifice in the bottom of the conduit, in relation to the increase in the flow rate outside the separator through the sharp-crested weir. For this reason, among the other things, calibration of the sharp-crested weir with a horizontal edge mounted in the conduit was the main study objective.

STUDY MATERIALS AND METHODS

FLOW DESCRIPTION

The sharp-crested weir placed across a circular cross-section of the pipe conduit in open channel flow conditions causes an increase of water depths upstream and flow over its crest, both for a sub- and super-critical flow (Fig. 3).

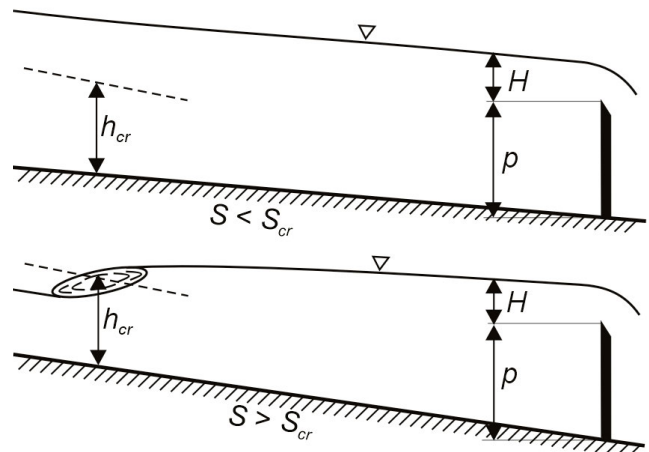


Fig. 3. Longitudinal water profile upstream the weir with slopes $S < S_{cr}$ and $S > S_{cr}$; S = channel bottom slope, S_{cr} = critical slope of the channel bottom; p = weir height, source: own elaboration based on Chow (1959)

Hager (1991) presented an explicit approach for backwater curves in circular pipes based on simple expressions for uniform and critical flows.

Troskoleński (ed.) (2014) distinguished three main types of the water flow over the sharp-crested weir in relation to the water head above the crest H (Fig. 4):

- a) adhering (clinging) nappe (Fig. 4a); the nappe developing when $H/p \approx 0.3$;
- b) nappe with suppressed bottom contraction; the space between the nappe and weir plate is filled with whirling (Fig. 4b);

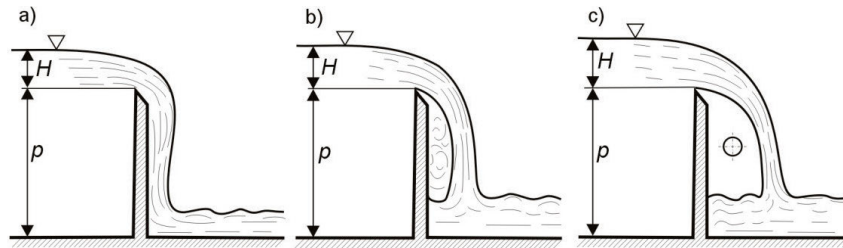


Fig. 4. Types of flow over the sharp crested weir; source: own elaboration based on Troskoleński (ed.) (2014)

c) free nappe; a nappe surrounded by air under atmospheric pressure, not wetting the downstream side of the weir (Fig. 4c).

Kindsvater and Carter (1951) investigated this type of weir in the rectangular channel and observed that the discharge conditions varied with the type of nappe profiles, while the upstream level was not influenced by the tail water elevation downstream the weir.

For the same water head above the crest H the discharge for the depressed nappe depends on the aeration, and the negative pressure is by 6–7% higher than for the free nappe. The discharge for the clinging nappe is 25 to 39% higher than the one with the free nappe. When the nappe is submerged, the discharge drops by 19%.

A flow through the sharp-crested weir in the pipe differs significantly from the flow through such weir in the river channel. The weir height in the channel is close to constant, while in the circular one it varies in the cross-section. In regions of small weir heights, i.e. at the contact of the weir and conduit walls, the nappe is drawn to the weir plate (Fig. 4a). In the middle part of the weir, where its height above the conduit bottom is the largest, the nappe detaches from (Fig. 4b) or clings to the weir plate (Fig. 4c). Conditions and types of flow over the weir in the pipe change along its crest.

The objective of the study is to calibrate the sharp-crested weir with horizontal edge within the pipe for the flow rate measurement. Due to the scale effect, stage-discharge relationships based on the calibration cannot be generalised to other pipe diameters and weirs heights than the one analysed.

THEORETICAL ANALYSIS

A functional relationship between the flow rate (Q) through the sharp-crested weir in the pipe conduit with open-channel flow conditions in a circular cross-section, a pipe diameter (D), a weir height (p), water head above the weir crest (H), a conduit slope (S), mean water velocity upstream the weir (v_o), water kinematic viscosity (ν), water density (ρ), a water surface tension (σ) and gravity acceleration (g), can be written after Rao (1975):

$$f(Q, D, p, H, v_o, \rho, \nu, \sigma, g, S) = 0 \quad (1)$$

Using a dimensional analysis for three-dimension independent variables (ρ, g, D) the following relationship for dimensionless variables is established:

$$\frac{Q}{\sqrt{gD^5}} = f\left(\frac{p}{D}, \frac{H}{D}, \text{Re}, \text{We}, \text{Fr}, S\right) \quad (2)$$

where: Fr, Re and We stands for pipe Froude, Reynolds and Weber numbers, respectively.

In studies on the discharge capacity of weirs, such hydraulic conditions are usually ensured and it is possible to eliminate the impact of viscosity and surface tension on the results. The elaborated discharge relations for weirs predict the discharge reasonably well at high heads. The effects of viscosity and surface tension of the liquid are important at low heads. Kindsvater and Carter (1951) and others concluded that these effects are negligible at higher values of water heads ($H > 0.03$ m). For rectangular and triangular weirs, Ranga Raju and Asawa (1977) determined that scale effects are negligible for Reynolds (Re) and Weber (We) numbers satisfying the condition $\text{Re}^{0.2}\text{We}^{0.6} > 900$. In practice, this condition corresponds to a head of 11 cm of water at 20°C.

However, it is not the case for weirs in a pipe. Due to relatively small dimensions, they operate also with lower water heads. The present investigations cannot be generalised to other pipe diameters than those analysed. Therefore, the weir discharge capacity was not described in terms of equation obtained by the energy principle. The relationship between the flow rate through the sharp-crested weir in the pipe conduit with open channel flow conditions was identified for dimensionless expression $\frac{Q}{\sqrt{gD^5}}$ and dimensionless variables: H/D , p/D and S .

EXPERIMENT

Hydraulic experiments determining the discharge capacity of sharp-crested weirs with horizontal edge in pipes with open-channel flow conditions were carried out in the hydraulic laboratory of the Faculty of Civil and Environmental Engineering, Warsaw University of Life Sciences. A discharge capacity of the sharp-crested weirs in the conduit with open channel flow conditions was analysed using a pipe with a circular cross-section made of a polyvinyl chloride (PVC) and an internal diameter of $D = 0.1534$ m (Fig. 5). The pipe was supplied from a cylindrical water tank of 1.5 m in diameter. The water tank was supplied from a laboratory-based closed water system. A water flow rate was measured by an induction flow meter. Slits of 20 mm were made at the upper side of the pipe enabling observations and providing ventilation. The weirs made of a steel plate with a horizontal crest placed across the pipe conduit were 1.5 mm in thickness and height of $p = 0.0465$ m, $p = 0.0634$ m and $p = 0.0771$ m. The weirs were not ventilated. Measurements were performed for two values of the conduit slope (S) 0.5 and 1.0%. These slopes were greater than the critical ones for the analysed flow rates ($S > S_{cr}$), thus water surface profiles upstream the weir were as shown in Figure 3. It also means that the flow downstream the weir was supercritical, so the downstream water level had no impact on the water level upstream the weir, and the weir was unsubmerged.

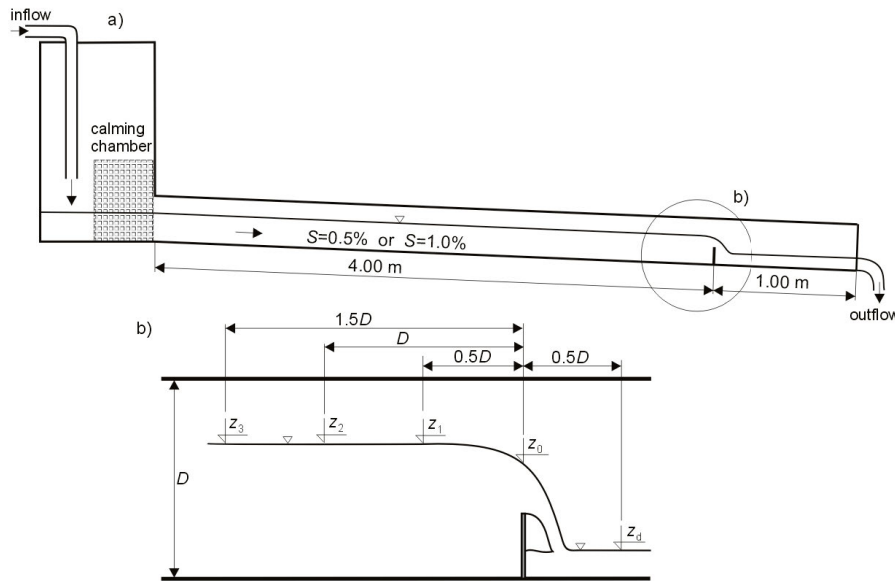


Fig. 5. Scheme of the pipe conduit with sharp-crested weir, showing measurement cross-sections; source: own elaboration

Water levels in the conduit were measured using a level gauge with an accuracy up to 0.1 mm. Directly, above the crest a water level z_0 was taken, z_1 in a distance of a half of the diameter, z_2 in one diameter and z_3 in one and a half of the diameter. Additionally, water temperature, maintained within the 14–16°C band, was recorded.

RESULTS AND DISCUSSION

EFFECT OF HEAD MEASUREMENT LOCATION

The first stage in the analysis of experimental results was to select a cross-section upstream the weir, for which measured water levels were used in the further analysis (Sokołowski, 2015). This was based on the calculated differences between measured levels upstream the weir: z_3-z_2 and z_2-z_1 . The obtained differences of water levels at cross-sections for the experiment with the weir of $p = 0.0465$ m in height and the conduit slope 0.5% are shown in Figure 6.

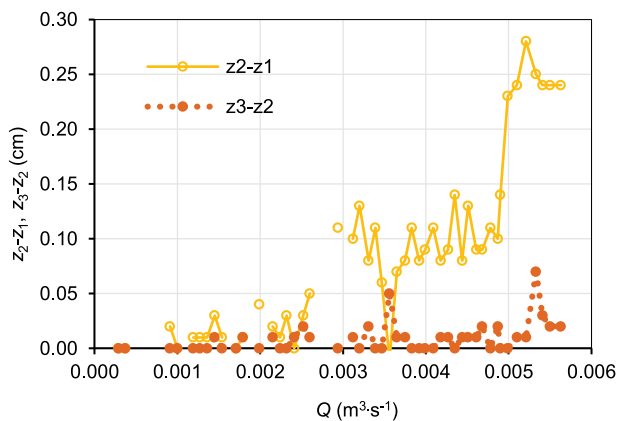


Fig. 6. Differences of the upstream water levels measured at cross-sections for sharp-crested weir of $p = 0.0771$ m in height for the conduit slope 0.5%; source: own study

As it is shown in Figure 6, differences of measured water levels upstream the weir z_3-z_2 are smaller than z_2-z_1 . The differences increase with the flow rate through the weir. Maximal differences of water levels for the outermost points at maximal discharge did not exceed 0.3 cm. This indicates that the curvature of the back-water curve at the cross-section $1.5D$ upstream the weir is small, and this cross-section was chosen as a representative for further analysis of the discharge capacity.

The measurements conducted with the conduit slope 0.5 and 1% showed that the back-water curve upstream the weir had a form presented in Figure 1b. The critical conduit slope at experimental flow rates was below 0.5%, which indicates super-critical flow conditions downstream the weir.

DISCHARGE-STAGE CURVE

An example of a discharge curve for the weir of a height of $p = 0.0771$ and conduit slope of 0.5% obtained based on measurements is given in Figure 7.

Green points in Figure 7 indicate the flow through the weir with the water stream adhering to the wall along the axis of the weir, i.e. flow forms shown in Figure 4a, b. The increase in the

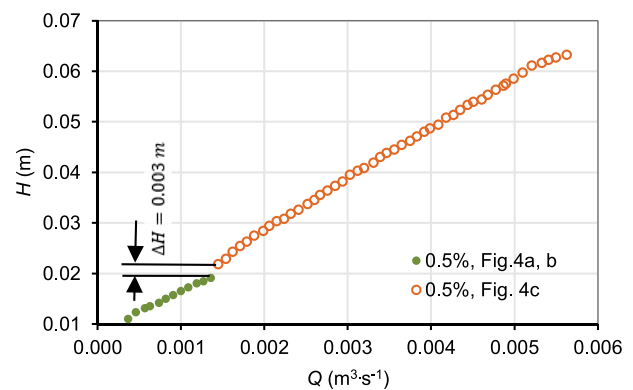


Fig. 7. Stage-discharge relationships for sharp-crested weir of $p = 0.0771$ m height for the conduit slope 0.5%; source: own study

flow rate above $0.0014 \text{ m}^3 \cdot \text{s}^{-1}$ resulted in stream detachment from the surface of the weir along the axis of the weir (orange points in Figure 7), aeration under the nappe and flow as shown in Figure 4c. As presented in Figure 7, a change in the form of the stream flow resulted in an increase in the height of the water (ΔH) overflowing over the horizontal crest of the weir by nearly 3 mm. A summary of the flow values (Q), which, when exceeded, caused the stream to become detached from the weir wall at different heights (p) and water height (ΔH) above the weir in the experimental variants is presented in Table 1.

Table 1. Water flow rates (Q), which, if exceeded, caused stream detachment from the weir wall in the axis of the conduit and increased the height of the water layer (ΔH) overflowing the horizontal edge of the weir

p (m)	S (%)	Q ($\text{m}^3 \cdot \text{s}^{-1}$)	ΔH (m)
0.0771	1.0	0.00136	0.0025
	0.5	0.00136	0.0027
0.0634	1.0	0.00111	0.0018
	0.5	0.00108	0.0026
0.0465	1.0	0.00074	0.0023
	0.5	0.00074	0.0032

Explanations: p = weir height, S = slope of the channel bottom.
Source: own study.

According to Unser and Holzke (1975), an increase in the water head ΔH for the flow through the weir with a sharp edge is similar to the capillary height at the vertical wall calculated as follows:

$$\Delta H = \sqrt{\frac{2\sigma}{\gamma}} \quad (3)$$

where: σ = water surface tension, γ = water specific weight.

For water at 15°C , the value of the capillary height at the vertical wall ΔH is equal to:

$$\Delta H = \sqrt{\frac{2 \cdot 0.07349 \frac{\text{N}}{\text{m}}}{9801.15 \frac{\text{N}}{\text{m}^3}}} = 0.00387 \text{ m} \approx 4 \text{ mm} \quad (4)$$

Figure 8 shows the limit values of the $\frac{Q}{\sqrt{gD^5}}$ ratios in the p/D function, the exceeding of which causes the stream to become detached from the weir wall in the axis of the conduit.

A dependence in Figure 8 shows that the water flow rate at which the stream detaches from the weir wall in a circular conduit of $D = 153.4 \text{ mm}$ in diameter increases with the weir height p and does not depend on the slope of the conduit S . The average increase in the height of the overflow layer ΔH , after detachment of the stream from the weir wall was approx. 0.003 m .

The heights of the water head layer above the edge of the rectangular weir in the channel, measured after the stream detaches from the weir wall with the free nappe, were compared with water head heights determined by Bazin (Smetana, 1948) for the sharp-crested weir (Fig. 9). According to Bazin's studies, for a high weir (in relation to the height of the water head), when the approaching stream velocity can be neglected, the ratio of the stream height in

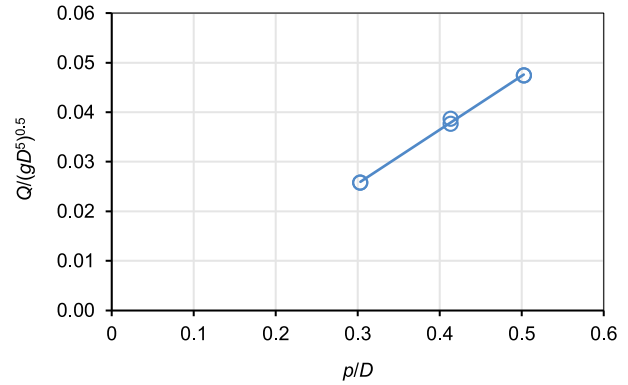


Fig. 8. The limit values of the $\frac{Q}{\sqrt{gD^5}}$ ratios in the p/D function, the exceeding of which causes stream detachment from the weir wall in the axis of the conduit; source: own study

the weir overflow section – H_p and the water head – H upstream the weir is equal to $H_p/H = 0.85$, as shown in Figure 9.

Changes in the ratio of the stream height in the weir overflow section to the height of the water head upstream the weir after nappe detached from the weir wall and the space under the stream become aerated (free nappe) for the examined weirs were compared with the value determined by Bazin for the sharp-crested weir (Fig. 9).

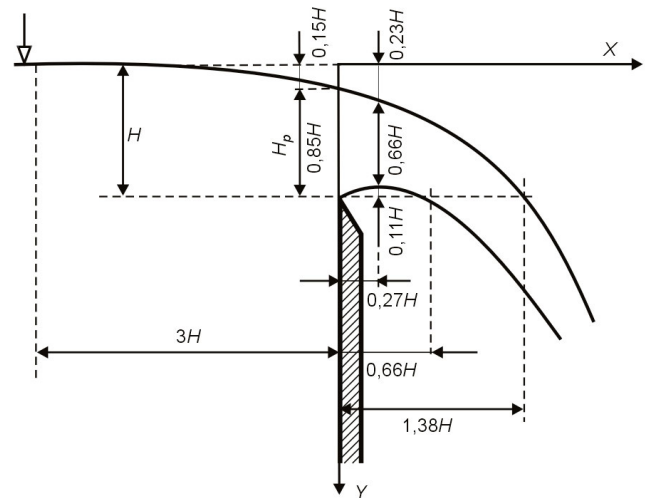


Fig. 9. Profiles of free nappe surfaces determined by Bazin for the water flow through high and sharp-crested weirs with aeration of the space under the nappe stream; source: own elaboration based on Kubrak (1981)

As can be seen from Figure 10, the stream height in the weir overflow section increases with the flow rate and decreases with the height of the weir. The increase in the conduit slope causes an increase in the stream height in the weir overflow section at the weir heights $p = 0.0634 \text{ m}$ and $p = 0.0771 \text{ m}$. With the weir height $p = 0.0771 \text{ m}$ and the conduit slope 0.5% , the values of this ratio approach the value of 0.85 , i.e. given by Bazin for rectangular sharp crested weirs in the rectangular channel. This means that even after the stream nappe detaches from the weir wall in the circular pipe, the stream profile does not correspond to the profile given by Bazin for sharp crested weirs. This statement is important because it means that the flow conditions and the discharge capacity of sharp crested weirs in circular conduits differ significantly from sharp crested weirs in channels.

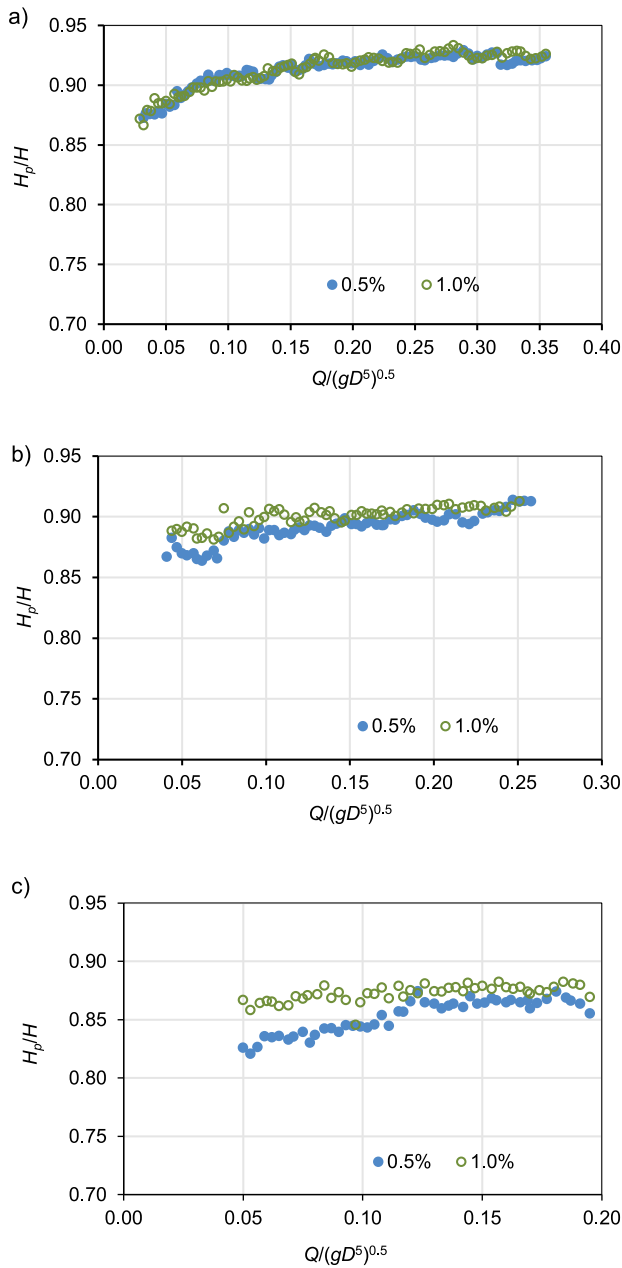


Fig. 10. Ratios of stream height in the weir overflow section to the height of the water head upstream the weir as a function of the water flow rate for the weir heights analysed; a) weir height $p = 0.0465$ m; b) weir height $p = 0.0634$ m; c) weir height $p = 0.0771$ m; source: own study

EFFECT OF PIPE SLOPE

Due to the described nature of changes in flow conditions and discharge capacity of weirs in circular pipes, it was found that discharge curves based on measurements of the weir flow with the free nappe, after the water stream detaches from the weir wall, can be used for measuring purposes, and these curves have been further analysed.

Discharge curves based on measurements for the weir height $p = 0.0465$ m with different conduit slopes are presented in Figure 11. Figure 11 shows that the discharge curves for the weir flow with the free nappe does not show a significant effect of the conduit slope on the weir discharge capacity.

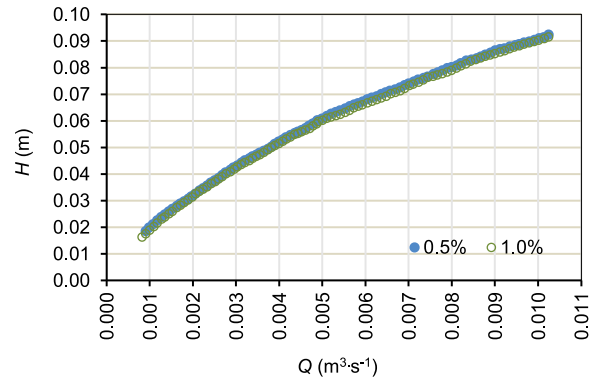


Fig. 11. Discharge curves for the weir with a height of $p = 0.0465$ m in a circular conduit with a diameter of $D = 0.1534$ m determined from measurements for the slopes of the conduit of $S = 0.5\%$ and $S = 1.0\%$; source: own study

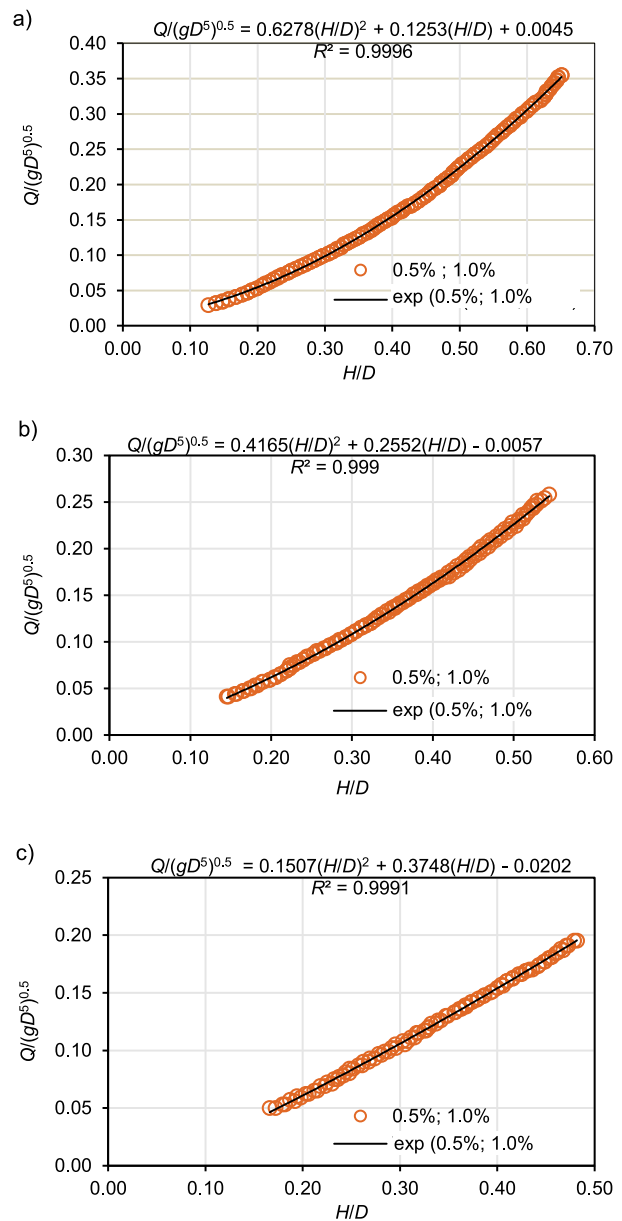


Fig. 12. Discharge curves for analysed weirs at conduit slopes of 0.5% and 1.0% after the water jet detaches from the wall of the weir; a) weir height $p = 0.0465$ m; b) weir height $p = 0.0634$ m; c) weir height $p = 0.0771$ m; source: own study

Therefore, the discharge curves for weirs of different heights based on measurements at conduits slopes of 0.5% and 1.0% after the water stream detaches from the weir wall is described by one dimensionless regression equation resulting from the relationship (2) and it is shown in Figure 12.

The regression equations summarised in Table 2 were fitted for the normalised discharge curves of the weirs.

The regression relationships Figure 12 with the coefficient of determination, standard deviation, and validity range are given in Table 2. As presented in Table 2 values of the determination coefficient and the standard deviation indicate that statistical dependency for the discharge is very strong and nearly mathematical.

value, it was noticed that conduit slopes had no effect on the shape of the discharge curves.

Ratios were determined for the analysed weir heights in the pipe of $D p/D \in (0.3; 0.4)$ and dimensionless values of $\frac{Q}{\sqrt{gD^5}}$. Larger ratio values cause the stream nappe to detach from the overflow wall. The detachment of the nappe from the weir wall was caused by a nearly 4 mm increase in the height of the water head upstream the weir. Based on the dimensional analysis, regression relationships for the weir flow rate in a pipe for different conduit slopes with distinction of the free nappe flow had a functional character and could be used to control the flow rate on the basis of a water head measured above the crest.

Table 2. General regression relationship for discharge capacity of investigated sharp crested weirs in the conduit of $D = 0.1534$ m

Weir height (m)	Regression equation, coefficient of determination R^2 , standard deviation δ_Q	Validity range
$p = 0.0465$ m $S = 0.5\%$ $S = 1.0\%$	$\frac{Q}{\sqrt{gD^5}} = 0.6278 \left(\frac{H}{D}\right)^2 + 0.1253 \left(\frac{H}{D} + 0.0045\right)$ $R^2 = 0.9996, \delta_Q = 0.00012 \text{ m}^3 \cdot \text{s}^{-1}$	$H \in (0.02 \text{ m}; 0.09 \text{ m})$ $Q \in (0.0008 \text{ m}^3 \cdot \text{s}^{-1}; 0.0102 \text{ m}^3 \cdot \text{s}^{-1})$
$p = 0.0634$ m $S = 0.5\%$ $S = 1.0\%$	$\frac{Q}{\sqrt{gD^5}} = 0.4165 \left(\frac{H}{D}\right)^2 + 0.2552 \left(\frac{H}{D}\right) - 0.0057$ $R^2 = 0.9990, \delta_Q = 0.00006 \text{ m}^3 \cdot \text{s}^{-1}$	$H \in (0.02 \text{ m}; 0.08 \text{ m})$ $Q \in (0.0010 \text{ m}^3 \cdot \text{s}^{-1}; 0.0075 \text{ m}^3 \cdot \text{s}^{-1})$
$p = 0.0771$ m $S = 0.5\%$ $S = 1.0\%$	$\frac{Q}{\sqrt{gD^5}} = 0.1507 \left(\frac{H}{D}\right)^2 + 0.3748 \left(\frac{H}{D}\right) - 0.0202$ $R^2 = 0.9991, \delta_Q = 0.00004 \text{ m}^3 \cdot \text{s}^{-1}$	$H \in (0.02 \text{ m}; 0.06 \text{ m})$ $Q \in (0.0014 \text{ m}^3 \cdot \text{s}^{-1}; 0.0056 \text{ m}^3 \cdot \text{s}^{-1})$

Explanations: p = weir height, S = slope of the channel bottom, H = water head, Q = flow rate.
Source: own study.

As already mentioned, Ranga Raju and Asawa (1977) determined for rectangular and triangular weirs that scale effects are negligible for Reynolds (Re) and Weber (We) numbers satisfying the condition $Re^{0.2}We^{0.6} > 900$. The values of $Re^{0.2}We^{0.6}$ obtained are in the range from 18 to 646, and do not meet the condition allowing to neglect the scale effect. Therefore, based on the calibration, the relationships cannot be generalised to cover other pipe diameters and weirs heights than those analysed because of the scale effects.

CONCLUSIONS

The results apply to specific physical models tested. There are three types of flow over sharp crested weirs in pipes. Sharp crested weirs in the pipe with open-channel flow conditions can be used in to control the flow for free nappe weir flows, after detachment of the stream along the axis of the conduit from the wall of the weir (Fig. 4c). The analysis of the water profile upstream the weir revealed that water level should be measured in a distance of one and a half pipe diameter upstream the weir. At this distance, the effect of the curvature of the depression curve, induced by the weir, becomes insignificant. For the conduit slopes that exceed the critical

Due to the scale effects, the relationships obtained from this calibration cannot be generalised to cover other pipe diameters and weirs heights than those analysed. Therefore, the effect of viscosity and surface tension on the weir effectiveness was not taken into account.

CONFLICT OF INTERESTS

All authors declare that they have no conflict of interests.

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